Types of Number

First things first, here are some **important terms** that will crop up **throughout A-Level Maths**. If you're clear on what they mean now, it'll make A-Level life a lot easier.

Integers are just Whole Numbers

An **integer** is any positive or negative **whole number** (including **zero**). The set of integers is represented by the symbol $\mathbb{Z}$.

Rational Numbers can be Written as Fractions

1) A **rational** number is any number which can be written as a **fraction** with integers on the top and bottom. Don't forget, any **integer** can be written as a **fraction over 1**.
2) The set of **rational numbers** is represented by the symbol $\mathbb{Q}$.
3) Both **terminating** and **recurring decimals** are rational.
4) A number which **cannot** be written exactly as a fraction is **irrational**. Irrational numbers are **non-repeating** decimals, which **never end**.
5) For example, π is a **never-ending**, **non-repeating** decimal ($\pi = 3.1415926535...$). This means that π is **irrational**.

The square roots of any non-square integer are irrational.

EXAMPLE: a) Is $0.\dot{6}$ a rational or irrational number?

$0.\dot{6}$ is the recurring decimal 0.666666... Recurring decimals are **rational** — $0.\dot{6} = \frac{2}{3}$.

b) Is 5.26 a rational or irrational number?

5.26 is **rational** because it is a terminating decimal $\left(5.26 = \frac{526}{100} = \frac{263}{50}\right)$.

All Rational and Irrational Numbers are Real Numbers

1) **Any rational** or **irrational** number is a **real number**.
2) The set of **real numbers** is represented by the symbol $\mathbb{R}$.
3) There are also numbers called **imaginary numbers.** They're **not real numbers**.
4) Imaginary numbers are the result of taking the **square root** of a **negative number**. It's **not possible** to do this with real numbers because the square of a real number is always positive, so **imaginary numbers** are used in these cases. You'll learn about them if you do A-Level Further Maths.

Rational? Irrational? Whatever man — I just keep it real...

1) Are these numbers rational or irrational? Explain your answer.
   a) 0.236849 b) $0.147\dot{8}$ c) $\sqrt{64}$ d) $\sqrt{3}$ e) 2π
2) True or false?
   a) All integers are rational. b) A recurring decimal is irrational.

Fractions

Multiplying Fractions — just Multiply the Numbers

1) Multiply the numerators and denominators **separately**.
2) Make sure to turn any **mixed numbers** into **improper fractions** first.
3) **Cancel** the fractions down **before** you start multiplying if you can — it'll make things easier.

EXAMPLE: Find $1\frac{3}{4} \times 2\frac{2}{3}$.

This is $1\frac{3}{4} = 1 + \frac{3}{4} = \frac{4}{4} + \frac{3}{4}$.

Write the mixed numbers as improper fractions: $1\frac{3}{4} \times 2\frac{2}{3} = \frac{4+3}{4} \times \frac{6+2}{3} = \frac{7}{4} \times \frac{8}{3}$

Now cancel down across your fractions and multiply.
8 divides by 4, so you can cancel these down. $\frac{7}{\cancel{4}_1} \times \frac{\cancel{8}^2}{3} = \frac{7}{1} \times \frac{2}{3} = \mathbf{\frac{14}{3}}$

Dividing by a Fraction — Flip It and Multiply

1) To **divide** by a fraction, just turn the fraction you're dividing by **upside down**, and **change** the divide sign to a **multiply**.
2) Just like before, turn **mixed numbers** into **improper fractions** and do any **cancelling** before you start.

EXAMPLE: Calculate $\frac{3}{8} \div \frac{7}{16}$.

Flip the second fraction and multiply. Make sure you do any cancelling you can before multiplying. 16 divides by 8, so you can cancel these down:

$\frac{3}{8} \div \frac{7}{16} = \frac{3}{\cancel{8}_1} \times \frac{\cancel{16}^2}{7} = \frac{3}{1} \times \frac{2}{7} = \mathbf{\frac{6}{7}}$

The Denominators need to Match for Addition and Subtraction

1) In order to **add** or **subtract** fractions, the **denominators** must be the **same**. So you have to find the **lowest common multiple** of all **denominators** (the lowest common denominator).
2) You still need to turn any mixed numbers into improper fractions.
3) Once both fractions have a common denominator, you can add/subtract their **numerators**.

EXAMPLE: Find $1\frac{5}{6} - \frac{3}{4}$.

You'll need to be confident with all of these rules as they're really important for using algebraic fractions.

Convert the mixed number to an improper fraction: $1\frac{5}{6} - \frac{3}{4} = \frac{11}{6} - \frac{3}{4}$.

Now they need a common denominator. The LCM of 6 and 4 is 12, so $\frac{11}{6} - \frac{3}{4} = \frac{22}{12} - \frac{9}{12}$

This means you can now subtract the numerators: $\frac{22}{12} - \frac{9}{12} = \mathbf{\frac{13}{12}}$

Denominators, much like socks, are best kept in matching pairs...

1) Calculate the following, giving your answers as improper fractions in their simplest form:

a) $2\frac{2}{3} \times \frac{1}{4}$ b) $5\frac{1}{3} \div 2\frac{1}{4}$ c) $\frac{3}{4} + \frac{1}{3}$ d) $1\frac{1}{6} + 2\frac{1}{2}$

e) $1\frac{3}{7} - \frac{2}{9}$ f) $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$ g) $\left(\frac{7}{6} - \frac{1}{7}\right) \div \frac{1}{2}$ h) $\frac{3}{2} - \frac{1}{9} + 2\frac{1}{3}$

Laws of Indices

Indices are Powers

For the value 4^3, 4 is the **base** and 3 is the **index**. (The plural of index is **indices**.)
You'll use indices all the time in A-Level Maths — especially for topics like **algebra**, **differentiation** and **integration**. So make sure you know the **laws of indices** inside out...

Multiplying Indices = Add the Powers, Dividing = Subtract them

So $\boldsymbol{x^a \times x^b = x^{(a+b)}}$ and $\boldsymbol{x^a \div x^b = x^{(a-b)}}$

EXAMPLE: a) Simplify $a^4 \times a^2$.

Just add the indices: $a^4 \times a^2 = a^{(4+2)} = \boldsymbol{a^6}$.
You can see how this works by rewriting the calculation as one big multiplication:
$a^4 \times a^2 = (a \times a \times a \times a) \times (a \times a) = a \times a \times a \times a \times a \times a = a^6$

b) Rewrite $4^5 \div 4^3$ as a single power of 4.

This time, subtract the indices: $4^5 \div 4^3 = 4^{(5-3)} = \mathbf{4^2}$
Again, you can see how this works by rewriting the calculation:
$4^5 \div 4^3 = \dfrac{\cancel{4} \times \cancel{4} \times \cancel{4} \times 4 \times 4}{\cancel{4} \times \cancel{4} \times \cancel{4}} = 4 \times 4 = 4^2$

Remember — this law only works when the two values have the same base. So $2^4 \times 2^7 = 2^{(4+7)} = 2^{11}$ is fine, but you can't use this rule to work out $2^4 \times 3^5$.

EXAMPLE: a) Simplify $8a^5 \times 2a^6$.

Both terms have the same base, a, and multiplication is **commutative** (which means it doesn't matter what order you do it in).
So you can rewrite this as: $8 \times a^5 \times 2 \times a^6 = (8 \times 2) \times (a^5 \times a^6) = 16 \times a^{(5+6)} = \boldsymbol{16a^{11}}$

b) Simplify $(x-1)^9 \div (x-1)^4 \times y^3 \times y^4$.

Deal with the powers of each base separately:
$(x-1)^9 \div (x-1)^4 \times y^3 \times y^4 = (x-1)^{(9-4)} \times y^{(3+4)} = \boldsymbol{(x-1)^5 y^7}$

Don't be put off by the brackets — $(x-1)$ is just the base.

There are Rules for x^0 and x^1

$x^0 = 1$ for any value of x The easiest way to see this is this with an example:
$4^3 \div 4^3 = 4^{(3-3)} = 4^0$ and $4^3 \div 4^3 = 1$, so $4^0 = 1$.

$x^1 = x$ for any value of x Again, this is easiest to understand as an example:
$4^3 \div 4^2 = 4^{(3-2)} = 4^1$, and $4^3 \div 4^2 = 64 \div 16 = 4$, so $4^1 = 4$.

Revise more I tell you — sorry all these powers have gone to my head...

1) Simplify the following:

a) $b^5 \times b^6$ b) $a^9 \times a \times b^5$ c) $c^5 \div c^2$ d) $9y^{10} \div 3y^{-2}$

e) $x^2 \times x^3 \div x^4$ f) $z^3 \times (y+2)^5 \times z \div (y+2)^2$ g) $a^{(x+2)} \times a^{2x}$ h) $x^{-2}y^5 \times x^5y^2$

Laws of Indices

To Raise a Power to Another Power, Multiply them

So $(x^a)^b = x^{ab}$

EXAMPLE: Simplify $(q^3)^2$.

$(q^3)^2 = q^{(3 \times 2)} = \boldsymbol{q^6}$

You can see how this works by rewriting it as one big multiplication: $(q^3)^2 = q^3 \times q^3 = (q \times q \times q) \times (q \times q \times q)$ $= q \times q \times q \times q \times q \times q = q^6$

EXAMPLE: Express $2^6 \div 4^2$ as a single power.

You need to make the bases the same before simplifying, so rewrite 4 as a power of 2 using the rule above: $2^6 \div 4^2 = 2^6 \div (2^2)^2 = 2^6 \div 2^{(2 \times 2)} = 2^6 \div 2^4 = \mathbf{2^2}$

Negative and Fractional Indices are a bit Trickier

A **negative** index means '**1 ÷ the positive power**' — so $x^{-a} = \dfrac{1}{x^a}$

For a fraction raised to a negative power, you turn the fraction upside down, then apply the positive index.

EXAMPLE: **a)** Write 3^{-4} as a fraction.

$3^{-4} = \dfrac{1}{3^4} = \mathbf{\dfrac{1}{81}}$

b) Write $\dfrac{1}{125}$ as a power of 5.

$125 = 5^3$, so $\dfrac{1}{125} = \dfrac{1}{5^3} = \mathbf{5^{-3}}$

If a number has a **fractional** index, this means '**the root of**' — so $x^{\frac{1}{a}} = \sqrt[a]{x}$

EXAMPLE: **a)** Find $64^{\frac{1}{3}}$ without using a calculator.

$64^{\frac{1}{3}} = \sqrt[3]{64}$

This is the cube root of 64, so the answer is **4**. (Because $4 \times 4 \times 4 = 64$.)

b) Find $\left(\dfrac{1}{256}\right)^{\frac{1}{4}}$ without using a calculator.

When you raise a fraction to a power, you raise the top and bottom to that power:

$\left(\dfrac{1}{256}\right)^{\frac{1}{4}} = \dfrac{1^{\frac{1}{4}}}{256^{\frac{1}{4}}} = \dfrac{\sqrt[4]{1}}{\sqrt[4]{256}} = \mathbf{\dfrac{1}{4}}$

You might have to **rewrite** the index. Use the fact that $x^{\frac{a}{b}} = \left(x^{\frac{1}{b}}\right)^a$, which is the same as $(x^a)^{\frac{1}{b}}$.

If you get a **negative fractional index**, use this fact to turn $x^{-\frac{a}{b}}$ into $\left(x^{\frac{a}{b}}\right)^{-1}$ or $(x^{-1})^{\frac{a}{b}}$.

EXAMPLE: Find $27^{-\frac{1}{3}}$ without using a calculator.

$27^{-\frac{1}{3}} = \left(27^{\frac{1}{3}}\right)^{-1} = \left(\sqrt[3]{27}\right)^{-1} = (3)^{-1} = \mathbf{\dfrac{1}{3}}$

I feel for you — as an editor I know just how bad rewriting an index is...

1) Simplify the following expressions:

a) $a^0 \div b^{-2}$ b) $(4^x)^x$ c) $3^m \times 9^2$ d) $\left(\dfrac{1}{a}\right)^2 \times a^{-3}$ e) $\left(\dfrac{1}{z^9}\right)^{\frac{1}{3}}$

2) Evaluate the following powers without using a calculator:

a) $4^{\frac{1}{2}}$ b) $27^{\frac{2}{3}}$ c) $2 \times 32^{\frac{3}{5}}$ d) $\left(\dfrac{125}{8}\right)^{\frac{1}{3}}$ e) $\left(\dfrac{25}{4}\right)^{-\frac{3}{2}}$ f) $\left(\dfrac{16}{9}\right)^{-\frac{3}{2}}$

3) Express $(5^{\frac{1}{4}})^2 \times (5^{\frac{2}{3}})^{-\frac{3}{4}} \div (5^{-1})^{-2}$ as 5^k, where k is an integer.

Multiplying Out Brackets

Double Brackets and Squared Brackets

These techniques are really useful for lots of A-Level topics — like surds and solving various types of equations.

1) When expanding **double brackets**, multiply each term in one set of brackets by each term in the other. If each set of brackets contains **two** terms, you can use the **FOIL** method to make sure you don't miss any terms:

   Multiply the **First** term in each set of brackets together, then the two **Outside** terms, then the two **Inside** terms and finally the **Last** terms.

2) Write out **squared brackets** as double brackets and then use the FOIL method to avoid making silly mistakes.

Don't make the mistake of just squaring the stuff inside the brackets. In general, $(a + b)^2 \neq a^2 + b^2$.

EXAMPLE: Expand and simplify $(2x - 4)^2$.

$$(2x - 4)^2 = (2x - 4)(2x - 4) = (2x \times 2x) + (2x \times -4) + (-4 \times 2x) + (-4 \times -4)$$
$$= 4x^2 - 8x - 8x + 16 = \mathbf{4x^2 - 16x + 16}$$

Whenever you multiply out brackets, make sure you simplify at the end — the x terms combine here.

Triple Brackets

1) You can think of **triple brackets** as **double brackets** multiplied by **one more** set of brackets.
2) So, use the **double bracket method** on two of the sets, then multiply the **result** by the **remaining set of brackets**.
3) You'll probably end up with **more than two terms** in your '**result**', so you won't be able to use the FOIL method for the last bit. Instead, you should **break up** the **shorter** set of brackets so that each of its terms is multiplied by the **second** set of brackets **separately**.
4) Then multiply out each of the new sets of brackets **one at a time**.

EXAMPLE: Write $(x + 5)(x - 1)(x + 2)$ as a cubic expression.

$$(x + 5)(x - 1)(x + 2) = (x + 5)(x^2 + x - 2) = x(x^2 + x - 2) + 5(x^2 + x - 2)$$
$$= (x^3 + x^2 - 2x) + (5x^2 + 5x - 10) = \mathbf{x^3 + 6x^2 + 3x - 10}$$

Brackets can have More Than Two Terms

At A-Level, you might be asked to multiply out sets of brackets with **more than two terms**. As usual, you need to multiply **every** term in the **first** set of brackets by **every** term in the **second**. The method for this is just the same as **stage two** of the **triple bracket method** above — multiply **each term** in the **shorter** set of brackets by the other set **separately**.

EXAMPLE: Expand and simplify the expression $(x^2 - 5x - 1)(x + y + 1)$.

$$(x^2 - 5x - 1)(x + y + 1) = x^2(x + y + 1) + (-5x)(x + y + 1) + (-1)(x + y + 1)$$
$$= (x^3 + x^2y + x^2) + (-5x^2 - 5xy - 5x) + (-x - y - 1)$$
$$= \mathbf{x^3 + x^2y - 4x^2 - 5xy - 6x - y - 1}$$

Quintuple brackets — just pack it all in and go home...

1) Multiply out each of these sets of brackets:
   a) $(y + 3)(y - 6)$ b) $(a - 3)(b + 4)$ c) $(p - 1)(p - 2q)^2$ d) $(s^2 + s + 2)(2s^2 - 2s + 4)$

Factorising

Factorising is Putting *Brackets Into an Expression*

Factorising means finding **common factors** that are in each term in an expression. You can take common factors outside a set of brackets to **rewrite** an expression. It's a handy skill that's used all the time in A-Level Maths, e.g. in **algebraic fractions** and **solving quadratics** and **cubics**.

1) You have to spot **common factors** in each term. These then come out to the **front** of the expression, and you put what's left in **brackets**.
2) Make sure that after you've finished factorising, any sets of brackets **can't be factorised** any more.

Check your answer by multiplying the brackets out again — if it's right, you'll end up back where you started...

EXAMPLE: a) Factorise $14x + 21xy$.

$14x + 21xy = \mathbf{7x(2 + 3y)}$

b) Factorise $16x^3 + 4x^2 - 4x$.

$16x^3 + 4x^2 - 4x = \mathbf{4x(4x^2 + x - 1)}$

At A-Level, you might see expressions where the common factor is a **set of brackets**.

EXAMPLE: Factorise $3x(x + 2) - 4(x + 2)$.

$(x + 2)$ appears in both terms of the expression, so it's a common factor:
$3x(x + 2) - 4(x + 2) = \mathbf{(x + 2)(3x - 4)}$

The *Difference Of Two Squares* is *One Square Minus Another*

Any expression of the form $a^2 - b^2$ is called the **difference of two squares**.
It can be **factorised** really easily using this result:

$$a^2 - b^2 = (a + b)(a - b)$$

This result is used all the time to solve quadratics and cancel down algebraic fractions, so you need to be able to spot when you can use it.

You can see why this works by multiplying out $(a + b)(a - b)$:
$(a + b)(a - b) = a^2 - ab + ba - b^2 = a^2 - ab + ab - b^2 = a^2 - b^2$.

EXAMPLE: a) Factorise $x^2 - 25$.

$a = x$ and $b = 5$, so $x^2 - 25 = \mathbf{(x + 5)(x - 5)}$

b) Factorise $4x^2 - 81$.

$4x^2 = (2x)^2$, so $4x^2 - 81 = \mathbf{(2x + 9)(2x - 9)}$

If the number isn't a square, you can write it as a **square root squared** (see the next page).

EXAMPLE: Factorise $x^2 - 3y^2$.

$3y^2 = (\sqrt{3}\,y)^2$, so using the difference of two squares: $x^2 - 3y^2 = \mathbf{(x + \sqrt{3}\,y)(x - \sqrt{3}\,y)}$.

X-Factorising — warbling live on telly to earn the public's affection...

1) Factorise the following expressions:

a) $20x^2 - 4x$ b) $8x^2y + 28xy^2$ c) $3\pi a^2 + 4\pi ab + 2\pi a$ d) $5x^2(x - 1) - 2x(x - 1)$

e) $x^2 - 9$ f) $9x^2 - 25$ g) $p^2 - 49q^2$ h) $v^2 - 7u^2$

Surds

Surds are the Square Roots of Non-Square Numbers

Irrational numbers that can be written as **roots** ($\sqrt{\ }$) are called **surds.**
You need to be able to deal with them when working with **quadratics**, **cubics** and **vectors**.

There are Rules for Manipulating Surds

1) $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$

2) $a = (\sqrt{a})^2 = \sqrt{a}\sqrt{a}$

Be careful — $\sqrt{a} + \sqrt{b}$ doesn't equal $\sqrt{a+b}$.

3) $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$

This is just the difference of two squares with surds.

4) $(a+\sqrt{b})(a-\sqrt{b}) = a^2 - a\sqrt{b} + a\sqrt{b} - (\sqrt{b})^2 = a^2 - b$

EXAMPLE: a) Simplify $\sqrt{27} + 5\sqrt{3}$.

$$\sqrt{27} + 5\sqrt{3} = \sqrt{9 \times 3} + 5\sqrt{3}$$
$$= \sqrt{9} \times \sqrt{3} + 5\sqrt{3}$$
$$= 3\sqrt{3} + 5\sqrt{3} = \mathbf{8\sqrt{3}}$$

b) Expand and simplify $(2+\sqrt{2})^2$.

Multiply out the brackets using FOIL:

$$(2+\sqrt{2})^2 = (2+\sqrt{2})(2+\sqrt{2})$$
$$= 4 + 2\sqrt{2} + 2\sqrt{2} + \sqrt{2}\sqrt{2}$$
$$= 4 + 4\sqrt{2} + 2 = \mathbf{6 + 4\sqrt{2}}$$

Surds in the Denominator should be Rationalised

If there's a surd on the **bottom** of a fraction, you need to **get rid** of it. To do this, you multiply the **top** and **bottom** of the fraction by an expression that will give a **rational number** in the denominator. This is called **rationalising the denominator**.

EXAMPLE: a) Rationalise the denominator of $\dfrac{2\sqrt{2}}{\sqrt{5}}$.

You need to multiply top and bottom by something that will make the denominator rational — $\sqrt{5}$ will work because $\sqrt{a}\sqrt{a} = a$:

$$\frac{2\sqrt{2}}{\sqrt{5}} = \frac{2\sqrt{2}\sqrt{5}}{\sqrt{5}\sqrt{5}} = \mathbf{\frac{2\sqrt{10}}{5}}$$

b) Show that $\sqrt{80} + \dfrac{25}{\sqrt{5}} = 9\sqrt{5}$.

$\sqrt{80} = \sqrt{16}\sqrt{5} = 4\sqrt{5}$ and

$\dfrac{25}{\sqrt{5}} = \dfrac{25\sqrt{5}}{\sqrt{5}\sqrt{5}} = \dfrac{25\sqrt{5}}{5} = 5\sqrt{5}$, so:

$$\sqrt{80} + \frac{25}{\sqrt{5}} = 4\sqrt{5} + 5\sqrt{5} = \mathbf{9\sqrt{5}}$$

Little Miss Muffet, she sat on a tuffet, multiplying her surds by $\sqrt{a}$...

1) Simplify the following: a) $2\sqrt{24} + 3\sqrt{96}$ b) $\dfrac{\sqrt{120}}{\sqrt{15}\sqrt{2}}$ c) $(1+\sqrt{x})^2 - 2\sqrt{x}$

2) By rationalising the denominator, show that $\dfrac{\sqrt{3}}{\sqrt{20}} = \dfrac{\sqrt{15}}{10}$.

3) One side of a square is $\sqrt{7} + \sqrt{12}$ cm long. Calculate the square's area.

Surds

You can Rationalise more Complicated Denominators

You'll need a slightly **different method** for rationalising more **complicated** denominators.

1) Just like before, you multiply the **top** and **bottom** of the fraction by the **same** expression.
2) The expression you need to multiply by is just the **denominator** with the **opposite sign** in front of the **surd**.

EXAMPLE: Rationalise the denominator of $\frac{3}{1+\sqrt{2}}$.

The bit you want to rationalise is $1+\sqrt{2}$, so multiply the top and bottom by $1-\sqrt{2}$:

$$\frac{3}{1+\sqrt{2}} = \frac{3(1-\sqrt{2})}{(1+\sqrt{2})(1-\sqrt{2})} = \frac{3-3\sqrt{2}}{1^2-(\sqrt{2})^2}$$

$$= \frac{3-3\sqrt{2}}{-1} = \mathbf{3\sqrt{2}-3}$$

This comes from the difference of two squares rule on the previous page: $(a+\sqrt{b})(a-\sqrt{b}) = a^2 - b$, with $a = 1$ and $b = 2$. You could also just multiply the brackets out.

This trick also works when there's a **number in front** of the **root** in the denominator.
Just choose an expression with the **opposite sign** in front of the number multiplied by the surd.

EXAMPLE: Rationalise the denominator of $\frac{3+\sqrt{5}}{3+2\sqrt{5}}$.

Although this looks harder, just do the same thing.
The denominator is $3+2\sqrt{5}$, so multiply the top and bottom by $3-2\sqrt{5}$:

$$\frac{3+\sqrt{5}}{3+2\sqrt{5}} = \frac{(3+\sqrt{5})(3-2\sqrt{5})}{(3+2\sqrt{5})(3-2\sqrt{5})}$$

$$= \frac{3^2-6\sqrt{5}+3\sqrt{5}-2(\sqrt{5})^2}{3^2-6\sqrt{5}+6\sqrt{5}-(2\sqrt{5})^2}$$

$$= \frac{9-3\sqrt{5}-2\times 5}{9-2^2\times 5}$$

$$= \frac{-3\sqrt{5}-1}{-11} = \mathbf{\frac{3\sqrt{5}+1}{11}}$$

Multiplying out these brackets is quite tricky, so take your time and don't skip any steps. See p.10 for more on multiplying out brackets.

But $\sqrt{b}$ surprised her, snuck into the divider, and ruined Miss Muffet's day...

1) Rationalise the denominators of the following:

a) $\frac{1}{1-\sqrt{5}}$ b) $\frac{\sqrt{10}}{4+\sqrt{40}}$ c) $\frac{1+\sqrt{7}}{5+\sqrt{7}}$ d) $\frac{2+2\sqrt{2}}{2-2\sqrt{2}}$

2) Using the fact that $\sin 60° = \frac{\sqrt{3}}{2}$, show that the exact value of $\frac{4}{1-2\sin 60°}$ is $-2-2\sqrt{3}$.

Solving Equations

Use the *6-Step Method* to *Solve Equations*

Lots of A-Level Maths requires you to be able to **solve equations**. The basic skills covered here crop up loads in **differentiation**, **trigonometry**, and solving **quadratics** and **cubics**.

To solve an equation, follow the **steps** below — you can **ignore** any steps you don't need. This method works for **any variable** — but if you're solving for x:

1) Remove any **fractions** by multiplying by the denominator(s).
2) **Multiply out** any **brackets** (see page 10).
3) **Collect** all the **x-terms** on one side and all the **number terms** on the other.
4) By **combining like terms**, **reduce** it to the form '**Ax = B**'.
5) Finally, **divide** both sides **by A**. This gives your answer 'x = '.
6) If you had 'x^2 = ' instead, **square root** both sides to end up with '$x = \pm$ '.

EXAMPLE: Solve $\frac{2x}{3} + \frac{4-x}{4} = 7x$.

1) First, get rid of the fractions. Multiply by 3 and then 4 to cancel the denominators:
$\frac{3\times4\times2x}{3} + \frac{3\times4\times(4-x)}{4} = 3\times4\times7x$, so $8x + 3(4-x) = 84x$
2) Next, multiply out the set of brackets: $8x + 12 - 3x = 84x$
3) Now collect the x-terms on one side of the equation and the number terms on the other: $12 = 84x - 8x + 3x$
4) Combine like terms: $79x = 12$
5) And divide by 79 to get $\boldsymbol{x = \frac{12}{79}}$

You don't need step 6 because there's no x^2 term...

Here are a couple more examples — both have a **squared** term to deal with, so you'll need to use **step 6** of the method above.

Remember, when you take the **square root** of a number the answer can be **positive** or **negative**, so the equation will have **two solutions**.

Be careful — the context of a question might mean only one of these answers will make sense.

EXAMPLE: a) Solve $3x(x + 4) = 12x + 6$.

Multiply out the brackets: $3x^2 + 12x = 12x + 6$

Simplify: $3x^2 + 12x - 12x = 6$

$\Rightarrow 3x^2 = 6$

Divide by 3: $x^2 = 2$

Take the square root: $\boldsymbol{x = \pm\sqrt{2}}$

b) Solve $5a = \frac{125}{a}$.

Multiply by a to get rid of the fraction: $5a^2 = 125$

Divide by 5: $a^2 = 25$

So, $a = \pm\sqrt{25} = \boldsymbol{\pm 5}$

What does a square Dalek say? x^2-term-inate...*

1) Solve each of the following equations:
   a) $4(2x - 3) = 7x$ b) $3(x + 14) = x + 12$ c) $\frac{b-7}{3} + \frac{b+1}{5} = -1$ d) $\frac{q(q+7)}{7} - q = 4 - q^2$
2) The sides of a square are x cm long. The area of $\frac{1}{4}$ of the square is 25 cm^2. How long are the sides?

*I'm so sorry.

Rearranging Formulas

*The **Subject** of a **Formula** is the Letter on its **Own***

1) If a formula has a **single letter** on one side of the equals sign, this letter is known as the **subject** of the formula. For example, y is the subject of the formula $\boldsymbol{y = mx + c}$.
2) Sometimes you'll need to **rearrange** a formula to make a **different** letter the subject.
3) You'll use this a lot in the **mechanics** sections of A-Level Maths, especially with **constant acceleration formulas**, so make sure you've got it learned.

*Use These Handy **7 Steps** to **Rearrange Formulas***

1) **Square** both sides to get rid of any **square roots**.
2) **Multiply** by the denominator(s) to get rid of any **fractions**.
3) **Multiply out** any **brackets**.
4) **Collect** all the **subject** terms together on one side and all **non-subject terms** on the other.
5) By **combining** like terms, **reduce** it to the form '**Ax = B**'. You might have to do some **factorising** to get it in the right form.
6) **Divide** both sides **by A** to give '$x =$ '.
7) If you've got '$x^2 =$ ', **square root** both sides to get '$x = \pm$ '.

This method is the same as the one for solving equations — so you've already had some practice at it.

Remember, x is the subject here. A and B could be numbers or letters (or a mix of both).

EXAMPLE: Make n the subject of the formula $\sqrt{\dfrac{n+3}{a}} = 2m$.

First, square both sides to get rid of the square root: $\dfrac{n+3}{a} = 4m^2$

Then multiply by a to get rid of the fraction: $n + 3 = 4am^2$

Finally, collect all the terms without an n on the RHS by subtracting 3: $\boldsymbol{n = 4am^2 - 3}$

If the subject appears **more than once** in the formula, then you'll need to do some **factorising**.

EXAMPLE: Make a the subject of the formula $a^2 + b^2 = (3 + a)(a - b)$.

There aren't any square roots or fractions, so go straight to multiplying out the brackets: $a^2 + b^2 = 3a - 3b + a^2 - ab$.

Gather all the terms containing a on one side, and everything else on the other side: $a^2 - 3a - a^2 + ab = -3b - b^2$.

This can be simplified to $-3a + ab = -3b - b^2$.

Take out the common factors a on the LHS and $-b$ on the RHS to give: $a(b - 3) = -b(b + 3)$

(b – 3) is the same as (–3 + b), just rewritten because it's neater.

Finish off by dividing both sides by $(b - 3)$ to get $\boldsymbol{a = \dfrac{-b(b+3)}{b-3}}$

Loyal subjects I doth command thee — moveth to yonder side of the '='...

1) a) Express a in terms of b, given that $b(a + 2) = 4$.
   b) Make c the subject of $f = \dfrac{9}{5}c + 32$.
2) a) Make x the subject of the formula $y = \dfrac{2x^2 - 3}{x^2 - 1}$.
   b) Make t the subject of $s = \dfrac{\sqrt{t+u}}{u}$